Published by Institute of Physics Publishing for SISSA/ISAS
Received: *March 19, 2004*
Accepted: *May 24, 2004*

Confinement, glueballs and strings from deformed AdS

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Abstract: We study aspects of confinement in two deformed versions of the AdS/CFT correspondence — the GPPZ dual of $\mathcal{N}=1^*$ Yang Mills, and the Yang Mills* $\mathcal{N}=0$ dual. Both geometries describe discrete glueball spectra which we calculate numerically. The results agree at the 10% level with previous AdS/CFT computations in the Klebanov Strassler background and AdS Schwarzchild respectively. We also calculate the spectra of bound states of the massive fermions in these geometries and show that they are light, so not decoupled from the dynamics. We then study the behaviour of Wilson loops in the 10d lifts of these geometries. We find a transition from AdS-like strings in the UV to strings that interact with the unknown physics of the central singularity of the space in the IR.

Keywords: Nonperturbative Effects, Confinement, Brane Dynamics in Gauge Theories, AdS-CFT and dS-CFT Correspondence.

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1. Introduction

The AdS/CFT correspondence [1]–[3] has led to the development of dual string theory (supergravity) descriptions of a variety of large N gauge theories. As originally stated the duality applied to gauge theories with maximal supersymmetry including, in the case of four dimensions, $\mathcal{N} = 4$ Super Yang Mills. An interesting task is then to try to move in the direction of gauge theories which are more phenomenologically applicable. This necessarily means looking at cases with less than the maximal supersymmetry of the original duality. Several examples of such dual geometries have been constructed including the non-supersymmetric AdS Schwarzchild geometry [4], and the $\mathcal{N} = 1$ geometries of Klebanov Strassler [5] and Maldacena Nunez [6]. It is now well understood how to compute the glueball bound state predictions [7] and Wilson loop behaviour [8] in such theories and these geometries show confining behaviour.

In this paper we will consider some of the simplest interesting deformations of AdS, which will provide results for glueballs and confining strings that test the systematic errors within such AdS approaches. The geometries we will study are asymptotically AdS with deformations in the interior that can be interpreted as including relevant (mass) operators in the gauge theory. Thus the theories are strongly coupled $\mathcal{N} = 4$ super Yang Mills in the UV but have less symmetry in the IR. The simplest method for constructing such deformations [10] is to use five dimensional $\mathcal{N} = 8$ gauged supergravity where the scalar fields act as source and vev for operators in the field theory. Lifting the solutions back to 10d [11] is somewhat involved but the lifts of the two geometries we will study are in the literature [12, 13] (in fact the $\mathcal{N} = 1^*$ lift is not quite complete).

The first of the geometries we study is the GPPZ $\mathcal{N} = 1^*$ solution [14] where one of the scalars of $\mathcal{N} = 8$ supergravity has a non-trivial profile corresponding to a mass term for three of the four fermionic fields of $\mathcal{N} = 4$ super Yang Mills. The solution is supersymmetric so the 6 scalars are also massive. The IR of the theory has just a gauge field and a gaugino. Glueballs in such geometries correspond to excitations of the dilaton field in the background. The potential for such fluctuations is a well [15] and therefore gives rise to discrete solutions, showing that the theory has a mass gap and is confining — here we numerically compute the solutions and find the mass spectrum. The results match to two significant figures the predictions [35] from the Klebanov Strassler [5] construction, which should describe a similar IR gauge theory. This encouragingly suggests that the UV completions of the theories, which are different, are unimportant to the glueball spectrum. In the GPPZ solutions one can also switch on a second scalar corresponding to a real gaugino condensate [14]. Previous analysis has suggested that the solutions with too large a condensate are unphysical. From the glueball analysis we confirm the existence of an upper limit on the gaugino condensate parameter since above it the glueball potential becomes unbounded from below. Within the physical solutions the value of the gaugino condensate only changes the glueball masses by of order 10%. It is also straightforward, following the analysis of [17], to calculate the masses of the bound states of $(\lambda\lambda)$ gauginos. Since there are a set of solutions with different gaugino condensate the gravity dual describes a moduli space and therefore we find a massless bound state plus excitations. The degeneracy of the vacua is a symptom of large N and is unfortunately unhelpful for comparison to finite N lattice computations [18] where there is no massless state. One might hope to compare our glueball results to future lattice results though. Finally we test the decoupling of the UV degrees of freedom by calculating the masses of the bound states of heavy fermions; we find they are of comparable size to the glueball masses so the decoupling is far from complete. This makes the insensitivity of the glueball masses to the UV physics all the more remarkable. In principle one should take into account mixing between these states and those we consider. Such an analysis can be found for the $\mathcal{N} = 1^*$ theory in [32]. Comparison to those results shows that the mixing again only effects the glueball masse at the 10% level.

Our second geometry is the Yang Mills* dual [13] which in five dimensions has a single scalar switched on, corresponding to an equal mass for all four fermion fields of $\mathcal{N} = 4$ super Yang Mills. Brane probing the 10d lift [13] shows that the six scalar fields also have a mass and therefore the IR of the theory is just a non-supersymmetric gauge field. We find the potential relevant for the glueball spectrum of the theory, show that it is a bounded well and compute the discrete glueball spectrum. The spectrum matches at the 10% level with previous estimates of the spectrum from AdS Schwarzchild [7]. This is again a surprising

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level of insensitivity to the UV completion since the AdS Schwarzchild geometry describes a 5d gauge theory in the UV whilst YM* returns to 4d $\mathcal{N}=4$ super Yang Mills. We also calculate the fermionic bound state masses and find they lie close to the glueball spectrum showing the limitations of the decoupling.

Although these deformed geometries are easy to construct and work with we must be careful because they are singular in the deep interior. Naively one would conclude that there is something badly amiss! However, in a similar construction of the $\mathcal{N}=2^*$ theory [19]–[23] the interior is also divergent. There brane probing has revealed that the singularity matches to the pole of the one-loop coupling of the gauge theory (the enhançon mechanism [24]). This is a possible interpretation of the singularities here too. It is also possible that the configurations are analogous to the Polchinski Strassler $\mathcal{N}=1^*$ construction [16] in which the core of the geometry contains some complicated fuzzy expansion of the D3 branes although identifying such an object from the geometry appears hard. Further one might expect that the strong coupling might trigger additional operators to switch on and these might act to smooth the singularity. This is the mechanism that leads to smooth geometries in the Klebanov Strassler and Maldacena Nunez geometries. Thus our geometries may well be incomplete. Nevertheless they do have the correct UV behaviour and conformal symmetry breaking. We might hope that typical strong coupling phenomena will be triggered by a large but finite gauge coupling and that the geometry very close to the singularity will therefore not play a role in the geometry's predictions. The glueball computations support this conclusion since the bounded well that controls their mass forms before the singularity is reached.

To investigate this further we also study Wilson loops in these geometries. The Yang Mills* 10d lift is the most complete so we concentrate on that case. A probe fundamental string in the geometry is associated with the interaction between a test quark anti-quark pair in the background gauge configuration [8]. In pure AdS the wider the quarks are separated the deeper the string penetrates into AdS. The action of the string looks like a constant (corresponding to the quark mass) plus a term determined by conformal symmetry that goes as $-1/L$ where L is the quark separation. In the Yang Mills* geometry the solutions of the equations of motion of the string behave very differently. Asymptotically the geometry is AdS so for quarks that are close the behaviour is the same. However, as the string samples further into the deformed space the Euler Lagrange equation solutions describe a second solution in which the quarks are closer together again. We show that this solution has higher action than the AdS-like solution. Naively there appears to be a maximum length string (we suggested this interpretation in the first version of this manuscript). To understand the solutions better we study the action of a set of curves linking two quarks at each fixed separation. In fact a simple sine wave configuration linking the quarks is sufficient to reveal the role of the different solutions, even though this set of curves only contains an approximation to the true solution of the Euler Lagrange equations. We find that the second solution is in fact a global maximum of the action. Strings that penetrate deeper into the core than that configuration have falling action and the string in fact wants to fall into the singularity. We find a phase transition as the quarks are separated where AdS-like strings are replaced by strings entering the core. This

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is unfortunate since it means the physics described by the geometry can not be completely described by just the supergravity. A stringy resolution of the singularity is needed. One can imagine that all might yet be well since if there is some fuzzy brane construction then the set up might be consistent with confinement by mechanisms similar to those suggested in the Polchinski Strassler $\mathcal{N} = 1^*$ set up [16]. Our knowledge is clearly incomplete though and this should be taken into account when considering the glueball spectrum results.

We begin in the next section 2 with a review of the formalism for constructing deformed AdS theories and looking at fluctuations about such geometries. In section 3 we will study the glueballs of $\mathcal{N} = 1^*$, and in section 4 the glueballs of Yang Mills*. In section 5 we turn to the analysis of Wilson loops in these geometries.

2. Deformations and fluctuations

We briefly review the formalism for constructing deformed AdS geometries in 5d supergravity and for studying the fluctuations about such backgrounds.

2.1 Deforming AdS

The AdS/CFT correspondence [1, 3] maps supergravity fields to operators in the field theory. The theories we consider here correspond to deformations of $\mathcal{N} = 4$ Super Yang Mills by mass terms for the four fermions of the theory. These operators correspond to scalar fields of the 5d supergravity theory. The dependence of the supergravity field on the radial AdS coordinate encodes information about the RG flow of the field theory operator.

Thus we look for solutions involving only scalar fields and allow the scalars to vary only in the radial direction of AdS_5 [10]. The other non trivial field is the metric

$$ds^2 = e^{2A(r)} dx^\mu dx_\mu + dr^2 \,, \tag{2.1}$$

where $\mu = 0, \ldots, 3$, r is the radial direction in AdS_5, and $A(r) \to r$ as $r \to \infty$ so that the space becomes AdS_5 at infinity. With an appropriate parametrization the scalar lagrangian can be written as $\mathcal{L} = \frac{1}{2}(\partial\lambda)^2 - V(\lambda)$ and the equations of motion for λ and A are [10]

$$\lambda'' + 4A'\lambda' = \frac{\partial V}{\partial \lambda} \,, \qquad 6A'^2 = \lambda'^2 - 2V \,. \tag{2.2}$$

For large r, where the solution will return to AdS_5 at first order and $\lambda \to 0$ and $V \to \frac{m^2}{2}\lambda^2$, only the first equation survives with solution

$$\lambda = a e^{-(4-\Delta) r} + b e^{-\Delta r} \tag{2.3}$$

a and b are constants, while the conformal dimension Δ of the field theory operator is related to the mass of the supergravity scalar by

$$M^2 = \Delta(\Delta - 4) \,. \tag{2.4}$$

a is interpreted as the source for a field theory operator and b as the vev of that operator, since e^r has conformal dimension one.

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If the solution retains some supersymmetry then the potential can be written in terms of a superpotential [33]

$$V=\frac{1}{8}\left|\frac{\partial W}{\partial\lambda}\right|^2-\frac{1}{3}|W|^2 \tag{2.5}$$

and the second order equations reduce to first order

$$\lambda^{'}=\frac{1}{2}\frac{\partial W}{\partial\lambda}\,,\qquad A^{'}=-\frac{1}{3}W\,. \tag{2.6}$$

2.2 Linearized fluctuations

The supergravity scalars characterised by a non zero b in (2.3) map to field theory operator expectation values. We can therefore look for linearized fluctuations of a given scalar in a background, corresponding to bound states in the field theory associated with that operator. We follow the analysis in [4, 7, 17]. We look for normalisable solutions of the linearized wave equation

$$\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu)\delta\Phi=\frac{\partial^2 V}{\partial\Phi\partial\Phi}\delta\Phi\,, \tag{2.7}$$

that behave as plane waves at the boundary

$$\delta\Phi=\psi(r)e^{-ikx}\,. \tag{2.8}$$

To determine the spectrum of the fluctuations, a standard procedure in AdS is to reduce the above equation to a solution of a Schroedinger problem [4, 7, 17]. Making the change of coordinates ($r\to z$)

$$\frac{dz}{dr}=e^{-A} \tag{2.9}$$

and rescaling

$$\psi\to e^{-3A/2}\psi\,, \tag{2.10}$$

equation (2.7) takes the Schroedinger form

$$\left(-\partial_z^2+U(z)\right)\psi(z)=M^2\psi(z)\,, \tag{2.11}$$

where

$$U=\frac{3}{2}A^{''}+\frac{9}{4}(A^{'})^2+e^{2A}\frac{\partial^2 V}{\partial\Phi\partial\Phi}\,. \tag{2.12}$$

Primes now denote differentiation with respect to z. If this potential takes the form of a bounded well then there are discrete solutions (with $k^2=-M^2$) for the linearized fluctuation indicating confinement of the fields into discrete bound states.

Since we will work throughout in the z coordinates, we re-express the results of the previous subsection in these new coordinates. The second order equations of motion for the background deformation become

$$\lambda^{''}+3\lambda^{'}A^{'}=e^{2A}\frac{\partial V}{\partial\lambda}\,,\qquad 6A^{'2}=\lambda^{'2}-2e^{2A}V\,. \tag{2.13}$$

In the UV ($z\to 0$) the behaviour is

$$z=-e^{-r}\,,\qquad e^{2A}=\frac{1}{z^2}\,,\qquad \lambda=a(-z)^{4-\Delta}+b(-z)^{\Delta}\,, \tag{2.14}$$

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where, again, a is interpreted as a source for an operator and b as the vev of that operator since z has conformal dimension -1. Finally the first order equations (2.6) now read

$$\lambda' = \frac{1}{2} e^{A} \frac{\partial W}{\partial \lambda}, \qquad A' = -e^{A} \frac{1}{3} W . \tag{2.15}$$

3. The $\mathcal{N} = 1^*$ geometry

The $\mathcal{N} = 1^*$ theory is $\mathcal{N} = 4$ super Yang Mills with equal mass terms for the three adjoint chiral superfields leaving just the vector multiplet massless. The UV of the theory is conformal whilst the IR behaves like $\mathcal{N} = 1$ Yang Mills generating a gaugino condensate dynamically in the weak coupling limit. At large N, where we will be working, the UV is also strongly coupled so the chiral multiplets can not be considered fully decoupled. The theory at large N has been studied in [34] and it has been shown to have a set of discrete vacua differentiated by the magnitude of the gaugino condensate which is real.

The gravity dual of this theory was found by GPPZ [14]. In the 5d supergravity theory one must identify the scalars dual to the the dimension 3 operators

$$\mathcal{O}_m = \sum_{i=2}^{4} \psi_i \psi_i, \qquad \mathcal{O}_\sigma = \psi_1 \psi_1 . \tag{3.1}$$

These corresponds to two scalars m and σ in the 10 of the SO(6) gauge symmetry of the supergravity theory (the global $\mathrm{SU}(4)_R$ symmetry of the gauge theory).

The GPPZ flow tuned by these two scalars preserves an $SO(3)$ flavour symmetry. The supergravity fields that are singlet under this residual $SO(3)$ can be organized in one gravity multiplet and two other hypermultiplets [12].

The potential for σ and m is [14]

$$V = -\frac{3}{8}\left[\cosh^2\left(\frac{2m}{\sqrt{3}}\right) + 4\cosh\left(\frac{2m}{\sqrt{3}}\right)\cosh(2\sigma) - \cosh^2(2\sigma) + 4\right], \tag{3.2}$$

and can be obtained from a superpotential

$$W = -\frac{3}{4}\left[\cosh\left(\frac{2m}{\sqrt{3}}\right) + \cosh(2\sigma)\right], \tag{3.3}$$

as expected since the solutions should maintain $\mathcal{N} = 1$ supersymmetry. We can then work with the first order equations

$$\frac{\partial \sigma}{\partial z} = -\frac{3}{2} e^{A} \sinh(2\sigma), \tag{3.4}$$

$$\frac{\partial m}{\partial z} = -\frac{\sqrt{3}}{2} e^{A} \sinh\left(\frac{2m}{\sqrt{3}}\right), \tag{3.5}$$

$$\frac{\partial A}{\partial z} = \frac{1}{2} e^{A} \left[\cosh\left(\frac{2m}{\sqrt{3}}\right) + \cosh(2\sigma)\right]. \tag{3.6}$$

Equations (3.4)–(3.6) can be solved analytically [14], and the result is a family of solutions parametrized by the boundary values of the fields as $z \to 0$ (UV)

$$m \to -az, \qquad \sigma \to -bz^3, \qquad A \to -\log|z| . \tag{3.7}$$

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From (2.14), m corresponds to a mass for three of the adjoint fermions and σ is the gaugino condensate. In this paper we do not need the analytic expression for the flows, so we will solve the first order equations numerically subject to the boundary conditions (3.7).

Solutions exist for all initial choices of a and b but the resulting geometries are singular. In [17] it was argued that only a subset of these flows should be considered physical. There it was required that the supergravity potential evaluated on the solution should be bounded from above. This condition restricts us to flows with initial conditions

$$\begin{aligned} b &\leq 3^{-3/2} a \\ &\approx 0.19 a \end{aligned} \tag{3.8}$$

The gravity dual therefore predicts a moduli space of vacua with varying real gaugino condensate up to some maximum value ($0.19m$) which is plausibly the $N \to \infty$ limit of the field theory result discussed above.

We will see further evidence for this bound on a shortly. For convenience in what follows we will set $a = 1$ — that is we set the scale of all mass operators in terms of the chiral multiplets' mass.

3.1 Glueballs

The mass spectrum of bound states can be obtained from fluctuations of the dual scalar fields by finding the eigenvalues of (2.11). We first calculate the mass spectrum of glueballs with quantum numbers $J^{PC} = O^{++}$ associated with the correlators $\langle \mathrm{Tr}\, F^2(x)\, \mathrm{Tr}\, F^2(y) \rangle$. The operator $\mathcal{O} = \mathrm{Tr}\, F^2$ is dual to the dilaton, so we need the Schroedinger potential (2.12) in this case. The dilaton does not contribute to the supergravity scalar potential, so that the Schroedinger potential for the glueballs is just

$$U = \frac{3}{2} A'' + \frac{9}{4} (A')^2 \,. \tag{3.9}$$

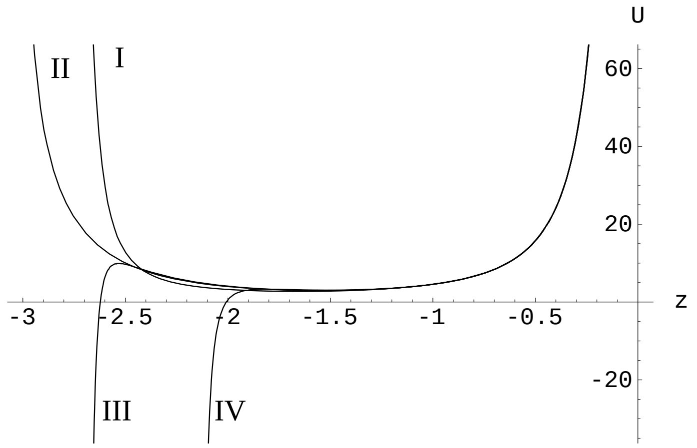

Figure 1: The glueball potential in $\mathcal{N} = 1^*$ with $a = 1$ and: I) $b = 0.1$; II) $b = 0.19$; III) $b = 0.2$; IV) $b = 0.25$.

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State	$\mathcal{N}=1^*$ $(b=0)$	$\mathcal{N}=1^*$ $(b=0.19)$	$\mathcal{N}=1$ KS	$(pL)^2 4(n+1)(n+4)$
0^{++}	1.0 (input)	1.0 (input)	1.0 (input)	1.0 (input)
0^{++*}	1.5	1.5	1.5	1.6
0^{++**}	2.0	1.9	2.0	2.1
0^{++***}	2.5	2.3	2.5	2.6
0^{++****}	3.0	2.7	3.0	3.2

Table 1: Spectrum of $\mathcal{N}=1^*$ glueball masses from supergravity with $b=0$ and $b=0.19$, not taking into account the mixing. In the next column the $\mathcal{N}=1$ glueball spectrum obtained from Klebanov Strassler background is shown for comparison. The last column shows the analytic result obtained in [32] taking into account mixing. In all cases the 0^{++} mass has been scaled to 1.

Actually the dynamics of the eight scalars of the full supergravity background is quite complicate and the dilaton, though being inert (that is having no radial dependence in the background solution) couples to the other scalars. To find the complete Schroedinger-like equation for the dilaton one has first to diagonalize the equation of motions for the system of scalars, as in [25], where an analytic solution has been found for all the scalars. Bound state spectra for some scalars had been previously calculated also in [26]–[29] and [32].

In order to obtain a discrete spectrum with a mass gap U must be bounded below. We plot the potential for varying b in figure 1. The potential is indeed bounded for all physical flows satisfying the bound (3.8). The unphysical flows with $b > 1.9a$, which violate (3.8), have an unbounded glueball potential. Thus we have further evidence for the bound.

The eigenvalues of (2.11), and hence the glueball mass spectrum, can be easily obtained using the numerical shooting method. Since we want the solution to match to the operator $\operatorname{Tr} F^2$ we set the UV $(z \to 0)$ boundary conditions on ψ to be

$$\psi(z) = \frac{1}{z^4}\,, \tag{3.10}$$

and then numerically solve for various M seeking regular solutions in the IR.

The results for $b=0$ and $b=0.19$ are shown in table 1. The 0^{++} mass is not a prediction, but just sets the strong coupling scale, so we normalize the lightest glueball state to 1. The gaugino condensate's value has little influence on the glueball masses (at most of order 10%).

We also quote the glueball mass spectrum obtained from the $\mathcal{N}=1$ Klebanov Strassler model [5, 35]. The Klebanov Strassler dual also describes $\mathcal{N}=1$ Yang Mills in the IR but in the UV there are extra flavours that participate in a cascade of Seiberg dualities. The glueball predictions agree very well with the $\mathcal{N}=1^*$ results, suggesting the UV completion of the theory is relatively unimportant to the glueball spectrum. The Maldacena Nunez [6] dual also describes $\mathcal{N}=1$ Yang Mills in the IR and the glueball spectrum has been recently analyzed in [36]. That dual though becomes strongly coupled in the UV so the computation is less clear. In [36] a UV cut off is imposed and then the glueball masses calculated — they are rather dependent on the choice of cut off. As noted by those authors the agreement with the Klebanov Strassler results is not so good.

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3.2 Gluino Bound States

We now move on to investigate the masses of bound states of the fermions of the $\mathcal{N} = 1^*$ theory. For simplicity we will set $\sigma = 0$ ($b = 0$) in what follows. To study bound states of the massless gluino we look at fluctuations of the scalar σ. The corresponding potential (2.12) is

$$U_\sigma = \frac{3}{2}A'' + \frac{9}{4}(A')^2 + 3e^{2A}\left[1 - 2\cosh\left(\frac{2m}{\sqrt{3}}\right)\right]. \tag{3.11}$$

State	ψ_1	ψ_2
1	0.0	1.2
2	1.0	1.7
3	1.6	2.2
4	2.1	2.8
5	2.7	3.3

Table 2: The first five bound states of the massless gluino ψ_1 and the massive fermion ψ_2 in the $\mathcal{N} = 1^*$ theory.

Numerically plotting this potential reveals it to be bounded giving a discrete spectrum with a mass gap. As in the case of the glueballs, we can find the eigenvalues of (2.11) with the shooting method. The results are shown in table 2 below, normalized to the lightest glueball mass above. The gravity dual describes a moduli space of vacua corresponding to different background values of σ so we would expect to find a massless boson. In fact we do and the smallest eigenvalue is zero.

Considerable work has been done by the DESY collaboration [18] studying pure $\mathcal{N} = 1$ Yang Mills on the lattice. They reach the supersymmetric theory by tuning the gaugino mass to zero and to date their results for bound state masses are still somewhat away from the supersymmetric point. In the future though it should be possible to extract the supersymmetric values and it would be nice to have AdS predictions available for comparison. The lattice theory at finite N has an anomalous $\mathrm{U}(1)_R$ symmetry and a complete mass gap. On the other hand in the $\mathcal{N} = 1^*$ theory that $\mathrm{U}(1)_R$ symmetry is only an accidental symmetry in the IR in the weak coupling limit and is not present at all in the large N theory [34] plus the gravity dual describes a massless bound state. The fermionic vacuum structure is therefore quite different and it is hard to compare the results for fermionic bound states. The best hope would be to compare the 0^{++} glueball spectrum above.

Finally it is also interesting to look at bound states of the massive fermions to see how decoupled these fields are from the dynamics. There is an SO(3) symmetry acting on the massive fermions so it is sufficient to analyse bound states of a single massive flavour. We therefore subdivide the operator $\mathcal{O}_m$ further introducing the scalars μ and ν corresponding to

$$\mathcal{O}_\mu = \psi_2\psi_2, \qquad \mathcal{O}_\nu = \sum_{i=3}^{4}\psi_i\psi_i\,. \tag{3.12}$$

This enables us to study bound states of the massive fermion ψ_2 by looking at fluctuations of μ. In terms of these scalars the potential is [21]

$$\begin{aligned} V = &\frac{1}{8}[-5 + \cosh(4\mu) - 4\cosh(2\mu)] - \cosh(\sqrt{2}\nu)[\cosh(2\mu) + 1] + \\ &+ \frac{1}{16}\left[-3 + 2\cosh(2\sqrt{2}\nu) + \cosh(4\mu)\right]. \end{aligned} \tag{3.13}$$

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Of course, we are still interested in preserving $\mathcal{N} = 1$ supersymmetry in the background. If we set

$$\nu = \sqrt{\frac{2}{3}} m\,, \qquad \mu = \frac{m}{\sqrt{3}} \tag{3.14}$$

then we recover the $\mathcal{N} = 1^*$ potential with $\sigma = 0$. The Schroedinger potential we are interested in for fluctuations in μ about this background is

$$U_\mu = \frac{3}{2}A^{''} + \frac{9}{4}(A^{'})^2 + e^{2A}\left[-2 - 2\cosh\left(\frac{2m}{\sqrt{3}}\right) + \cosh\left(\frac{4m}{\sqrt{3}}\right)\right]. \tag{3.15}$$

This is indeed a bounded potential. The mass spectrum is found by shooting and the results are displayed in table 2. We find that the fermion bound states are a little heavier than the lightest glueball but far from completely decoupled.

Since these states are not decoupled we must worry about mixing between them and the states whose masses we have calculated above. In [29, 32, 25] the authors make progress in this direction by calculating the spectra through the analysis of the two point correlation function for the associated operator, using the holographic renormalization method [30, 31, 32]. The glueball spectrum associated to the dilaton in the $\mathcal{N} = 1^*$ background was found analytically in [25] solving the full equations of motion, that is taking into account the mixing of the dilaton with other scalars in the supergravity theory. The authors found a spectrum given by $m^2 = (pL)^2 4(n+1)(n+4)$, with $n = 0, 1, 2, \ldots$. This spectrum, indicated in Table 1 above, is consistent at the 5-10% level accuracy with our numerical result. Our results for the gaugino bound state agree perfectly with their analytic expression for the spectrum $m^2 = (pL)^2 4(n+1)(n+2)$ with $n = 0, 1, 2, \ldots$. It is remarkable that the physics of the model is not dramatically affected by the mixing, and supports a more naive analysis where these extended super-partner states are neglected.

It is interesting to note that if we study fluctuations of the supergravity scalar m which corresponds to a composite operator of all three fermions (3.1) we obtain the potential

$$U_m = \frac{3}{2}A^{''} + \frac{9}{4}(A^{'})^2 + e^{2A}\left[-2\cosh\left(\frac{2m}{\sqrt{3}}\right) - \cosh\left(\frac{4m}{\sqrt{3}}\right)\right]. \tag{3.16}$$

This potential is unbounded. This presumably corresponds to an instability in the field theory for such a bound state to decay to the states we have seen above.

4. The Yang Mills* geometry

The Yang Mills* theory [13] is $\mathcal{N} = 4$ super Yang Mills with mass terms for all the adjoint matter fields leaving just non-supersymmetric gauge fields in the IR. At large N the massive fields can again not be considered totally decoupled.

The Yang Mills* geometry is obtained by turning on a supergravity scalar λ from the 10 of SO(6) dual to the operator

$$\mathcal{O} = \sum_{i=1}^{4} \psi_i \psi_i\,. \tag{4.1}$$

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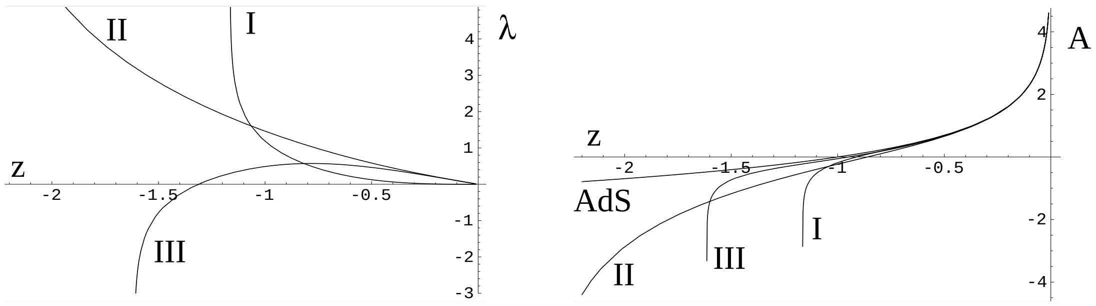

Figure 2: Plots of λ and A vs z in the YM* set up for a variety of UV initial conditions: I) condensate only ($\lambda \simeq -z^3$); II) mass only ($\lambda \simeq -z$); III) mass and condensate ($\lambda \simeq -(z+z^3)$). The AdS function A is also plotted.

The potential for the scalar can be obtained from the $\mathcal{N}=1^*$ solution [14] by setting the two scalars equal:[1]

$$V = -\frac{3}{2}\left(1+\cosh^2\lambda\right). \tag{4.2}$$

In this case $M^2=-3$ in (2.4) and the ultra-violet solutions are

$$\lambda = \mathcal{M}z + \mathcal{K}z^3 . \tag{4.3}$$

The field theory mass term has dimension 3. Thus in what follows $\mathcal{M}=0$ corresponds to a solution with just bi-fermion vevs while $\mathcal{K}=0$ corresponds to the purely massive case. Giving a mass to all four fermions breaks supersymmetry completely so we would expect the scalars to radiatively acquire masses so that the deep infra-red should be pure Yang Mills. The analysis of the 10d lift in [13] supports this hypothesis.

4.1 Numerical solutions

We are not able to write down first order equations for the deformations as there is no supersymmetry and no superpotential. Thus we are forced to solve the second order equations numerically. Numerical solutions of the flow equations for the scalars $\lambda(z), A(z)$ are displayed in figure 2 for different asymptotic boundary conditions. The mass only flow is unique, lying on the border between two separate behaviour flows. If there is even a small condensate $\lambda(z)$ diverges very rapidly. It is necessary to fine tune the initial conditions to high precision in order to isolate the mass only flow. All these flows are singular including the mass only solution.

We need a criteria for deciding which of these flows is the physical flow for the YM* theory. The simplest criteria we have found is the boundedness of the 0^{++} glueball Schroedinger potential. Thus we again look at linearized dilaton fluctuations and evaluate the Schroedinger potential in (3.9). We plot the potential for a variety of flows with

[1] To be precise one must set $m=\sqrt{3/4}\lambda$ and $\sigma=\sqrt{1/4}\lambda$ to maintain a canonically normalized kinetic term

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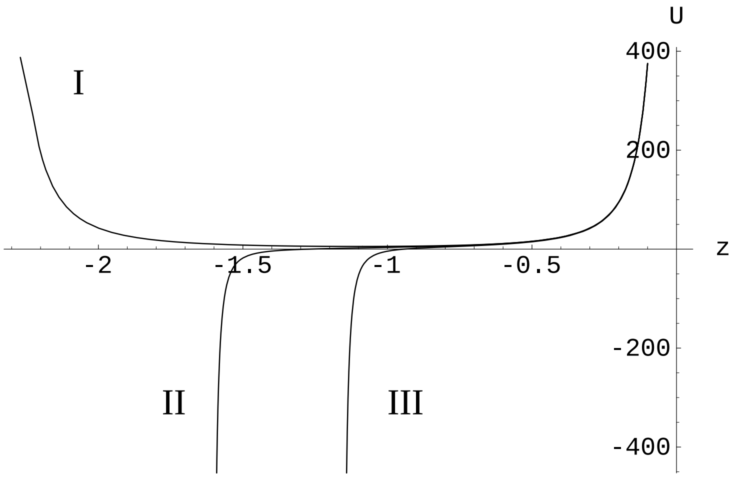

Figure 3: The glueball potential showing: I) mass only; II) mass and condensate; III) condensate only.

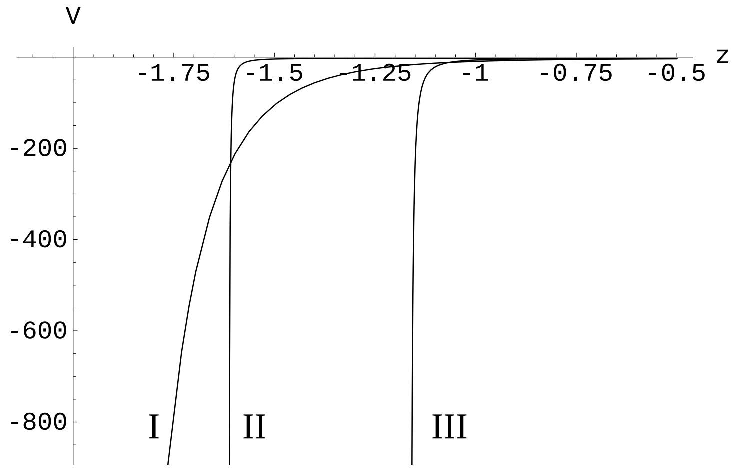

Figure 4: The supergravity potential for a range of flows: I) mass only; II) mass and condensate; III) condensate only. All cases are bounded above.

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both mass and condensate present in figure 3. The presence of a condensate leads to an unstable geometry, and an unbounded glueball potential. For the mass only solution, however, the glueball potential is a bounded well and the spectrum is calculable. We therefore conclude that this is the physical flow.

In [17] it was proposed that a necessary condition for singular flows to be physical is that the supergravity potential, evaluated on the solution of the equations of motion, should be bounded above. This condition is the origin of the bound (3.8). Figure 4 shows the supergravity potential evaluated for different asymptotic boundary conditions. All the Yang Mills* flows satisfy this condition, including the unphysical flows with a condensate. It is also a necessary condition for a confining gauge theory that the glueball potential be a bounded well. Here this appears to be a stricter condition that successful distinguishes the physical from the unphysical flows.

4.2 IR Asymptotic Solutions

We have also been able to analytically isolate the IR behaviour of the unique YM* flow that gives rise to a bounded glueball potential. In the r coordinates, the infra-red corresponds to $r \to r_0$, $\lambda(r) \to \infty$ and $A(r) \to -\infty$. The flow equations become

$$\lambda'' + 4A'\lambda' = -\frac{3}{4}e^{2\lambda}, \qquad 6A'^2 = \lambda'^2 + \frac{3}{4}e^{2\lambda}, \tag{4.4}$$

and have solutions

$$\lambda = \log\left(\frac{c}{|r-r_0|}\right), \qquad c = \sqrt{\frac{20}{9}}, \qquad A = \frac{2}{3}\log|r-r_0|, \tag{4.5}$$

where r_0 marks the position of the singularity. This is clearly not a complete set of solutions, so could represent any of the solutions in figure 2. To check whether this flow really corresponds to the solution with just a mass term, we can compute the glueball potential. This can easily done by changing variables $dz = e^A dr$

$$\begin{aligned} z &= \int e^{-A} dr \\ z - z_0 &= 3(r-r_0)^{1/3}. \end{aligned} \tag{4.6}$$

Hence

$$A(z) = 2\log|\frac{z-z_0}{3}|, \tag{4.7}$$

and the glueball potential $U(z)$ in the infra-red is given by

$$\begin{aligned} U(z) &= \frac{3}{2}A'' + \frac{9}{4}(A')^2 \\ &= \frac{6}{(z-z_0)^2}. \end{aligned} \tag{4.8}$$

We see that $\mathrm{U}(z) \to \infty$ in the infra-red. Comparison with numerical study of the full second order equations implies that these solutions are the IR limit of the mass only Yang Mills* geometry. If there had been a condensate present, we would have found $\mathrm{U}(z) \to -\infty$.

State	Yang Mills*	AdS-Schwarz	Lattice ($N=3$)	Lattice ($N=\infty$)
0^{++}	1.0 (input)	1.0 (input)	1.0 (input)	1.0 (input)
0^{++*}	1.5	1.6	1.5	1.9
0^{++**}	1.9	2.1	-	-
0^{++***}	2.3	2.7	-	-
0^{++****}	2.7	-	-	-

Table 3: Spectrum of glueball masses from the lattice, from Yang Mills* and Witten's AdS-Schwarzchild dual. Again, the lowest glueball mass has been scaled to one in all cases.

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4.3 Glueballs

As discussed above, the 0^{++} glueball Schroedinger potential for the mass only case is a bounded well. We calculate the glueball mass spectrum by shooting, and the results are shown in table 3. As in the previous example, the lightest glueball state is not a prediction, but can be used to fix the scale Λ_{QCD}. As in the $\mathcal{N} = 1^*$ case we normalize the lowest state to 1.

Again a possible source of inaccuracy in the nuerical results can derive from the mixing of the dilaton with other scalars. Even if there is no analytic proof for such mixing, we expect it to occur by analogy with the supersymmetric case.

An AdS approach to non-supersymmetric Yang Mills glueballs has already been presented in the literature. The AdS Schwarzchild black hole background describing M5 branes with a compact time dimension describes the high temperature limit of a strong coupled 5d field theory [3, 4]. The effect of the temperature is to give masses to all fields except the spatial components of the gauge field. Thus in the deep IR this theory is a 4d Yang Mills theory. This geometry was used to compute the glueball spectrum in [7] and we display the results in table 2 for comparison (an alternative calculation of the same spectrum is given in [37]). The results are surprisingly close, differing only at the 10% level. We believe that taking into account the mixing would increase the masses, as happens in the supersymmetric case, making the agreement even more good and more remarkable. Our theory is 4d $\mathcal{N} = 4$ Yang Mills in the UV whilst the AdS Schwarzchild dual describes a higher dimension theory in the UV. Since the UV is not decoupled from the strong interactions we might have expected at least order one differences. The closeness of the results hints at the relative stability of glueball masses across a wide range of theories and strengthens the case for trusting the AdS method of computation.

It is also interesting to compare the results to lattice computations in pure Yang Mills. Data is only available for the lightest two 0^{++} states. In Table 3 we show the data for $N = 3$ [38] and also the $N \to \infty$ extrapolation of [39]. The lattice errors are of order 10%. The AdS results actually match better to the $N = 3$ results but still agree with the $N \to \infty$ result at the 15% level. The data is very limited but again the agreement is surprising given that the UV of these theories are all so different.

4.4 Fermion bound states

To test the degree to which the massive fermions of the YM* theory have decoupled we will calculate the mass spectrum of bound states for them. We use the $\mathcal{N} = 1^*$ potential in (3.2) where the scalar σ describes the bi-fermion operator of a single flavour of adjoint fermions. There is again an SO(4) symmetry on the massive fermions so it is sufficient to study one flavour. Setting the fields m, σ to their YM* background values we then look at fluctuations of σ. The appropriate potential (2.12) for this scalar in the Yang Mills* background is

$$U_\sigma = \frac{3}{2}A'' + \frac{9}{4}(A')^2 - 3e^{2A}\,. \tag{4.9}$$

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State	mass
1	0.9
2	1.3
3	1.7
4	2.1
5	2.6

Table 4: First five bound states of a massive adjoint fermion in Yang Mills*.

This is again a bounded well potential and the spectrum of bound states is calculable. The results are shown in table 4. The bound states have masses of order the glueball masses and so can not be considered truly decoupled. Again this suggests that the success of the AdS computations of glueball masses to match lattice data above indicates the stability of the glueball masses across a range of gauge theories rather than that the decoupling of extra fields is being achieved.

5. Wilson loops

The quark anti-quark interaction potential may be studied in AdS duals [8] by introducing a probe D3 brane into the geometry at some radius z_{max}. Fundamental strings between the probe and the central stack of D3 branes would represent W bosons which transform in the $(N, 1)$ of the $\mathrm{SU}(N) \times \mathrm{U}(1)$ gauge group — we may equally think of these states as quarks since they are in the fundamental representation of $\mathrm{SU}(N)$. Thus a string attached to the probe with well separated ends play the role of a quark anti-quark pair with mass of order of the energy scale determined by z_{max}. The action of the string corresponds to the interaction energy between the pair. To study such a configuration we necessarily require the 10d lifts of our deformed geometries since the string lives in 10d. The lift of the Yang Mills* geometry has been found in [13] while for the $\mathcal{N} = 1^*$ geometry only a partial lift exists [12]. For these reasons we will work mainly with the Yang Mills* solution. Since the warp factor of the $\mathcal{N} = 1^*$ theory behaves similarly to that of Yang Mills* it is probable that the two theories have qualitatively the same Wilson loop behaviour.

The 10d lift of YM* is

$$ds_{10}^2 = (\xi_+\xi_-)^{1/4} ds_{1,4}^2 + (\xi_+\xi_-)^{-3/4} ds_5^2 \,, \tag{5.1}$$

where $ds_{1,4}^2$ is given in (2.1) and

$$ds_5^2 = \xi_- \cos^2\alpha \, d\Omega_+^2 + \xi_+ \sin^2\alpha \, d\Omega_-^2 + \xi_+\xi_- d\alpha^2 \,. \tag{5.2}$$

The functions $\xi_\pm$ are given by

$$\xi_\pm = c^2 \pm s^2 \cos 2\alpha \,, \qquad c = \cosh\lambda \,, \qquad s = \sinh\lambda \,. \tag{5.3}$$

The dilaton is given by

$$e^{-\Phi} = \sqrt{\frac{\xi_-}{\xi_+}} \,, \tag{5.4}$$

The two-form potential is given by

$$A_{(2)} = iA_+ \cos^3\alpha \, \cos\theta_+ d\theta_+ \wedge d\phi_+ - A_- \sin^3\alpha \, \cos\theta_- d\theta_- \wedge d\phi_- \,, \tag{5.5}$$

with $A_\pm = \sinh 2\lambda / \xi_\pm$. Finally the four-form potential lifts to

$$F_{(4)} = F + \star F \,, \qquad F = dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3 \wedge d\omega \,, \tag{5.6}$$

where $\omega(r) = e^{4A(r)} A'(r)$.

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We can calculate the Wilson loop behaviour by lying a string in an $x_{//}$ direction and letting it move in r and α. The action for the string in Einstein frame is given by

$$S = \frac{1}{2\pi\alpha'} \int d^2\sigma e^{\Phi/2} \sqrt{\det G} \,. \tag{5.7}$$

With this choice of orientation, the fundamental string does not couple to the background B-field (5.5) which has non-zero components only on the internal manifold. In the static gauge ($\sigma = x^1 = x$, $\tau = x^0$) and using the z coordinates ($ds^2 = e^{2A(z)}(dx^2 + dz^2)$) we obtain

$$S = \frac{T}{2\pi} \int dx \, e^{2A} \sqrt{\xi_+} \sqrt{1 + \left(\frac{dz}{dx}\right)^2 + e^{-2A}\left(\frac{d\alpha}{dx}\right)^2} \,, \tag{5.8}$$

where T is the period resulting from the time integration. The resulting equations of motion for z and α are then

$$\begin{aligned} \frac{d}{dx}\left[\frac{e^{2A}\sqrt{\xi_+}z'}{\sqrt{1+z'^2+e^{-2A}\alpha'^2}}\right] - \frac{d}{dz}\left[e^{2A}\sqrt{\xi_+}\sqrt{1+z'^2+e^{-2A}\alpha'^2}\right] &= 0 \,, \\ \frac{d}{dx}\left[\frac{e^{2A}\sqrt{\xi_+}\alpha'}{\sqrt{1+z'^2+e^{-2A}\alpha'^2}}\right] - \frac{d}{d\alpha}\left[e^{2A}\sqrt{\xi_+}\sqrt{1+z'^2+e^{-2A}\alpha'^2}\right] &= 0 \,. \end{aligned} \tag{5.9}$$

These can be solved numerically but there are an array of solutions. The new feature relative to pure AdS is that the strings can have a non-trivial α profile. The reason for this is that the fermion masses we introduced break the SO(6) symmetry to $SO(3) \times SO(3)$ so there are potentially different strings connecting particles in different subgroups of the SO(6). To begin with lets concentrate on strings that are between two identical particles and hence have no α variation. There are such strings — in the α EoM the last potential term is given by

$$-\frac{d\sqrt{\xi_+}}{d\alpha}\left[e^{2A}\sqrt{1+z'^2+e^{-2A}\alpha'^2}\right] = -\frac{\sinh^2\lambda \sin 2\alpha}{\sqrt{\xi_+}}\left[e^{2A}\sqrt{1+z'^2+e^{-2A}\alpha'^2}\right], \tag{5.10}$$

so vanishes if $\alpha = n\pi/2$. The kinetic term also vanishes if α =constant so these are solutions of the equations of motion.

As a simple case we study strings for which $\alpha = \pi/2$ which implies $\xi_+ = 1$. The z EoM then becomes

$$\frac{d}{dx}\left[\frac{e^{2A}z'}{\sqrt{1+z'^2}}\right] - \frac{d(e^{2A})}{dz}\sqrt{1+z'^2} = 0 \,. \tag{5.11}$$

This is straightforward to solve numerically in the background $A(z)$ appropriate to YM* (shown in figure 2).

Consider initial conditions where we start the string at $z = -0.5$, which is in the AdS like region. We then vary the derivative of z' and shoot off strings to more negative z corresponding to the interior of the deformed AdS space — we show numerical results of this type in figure 5. Initially as z' increases the strings penetrate the geometry more and return to $z = -0.5$ at larger x indicating that they describe a quark anti-quark pair that are more widely separated. This is standard AdS behaviour. However when the strings begin

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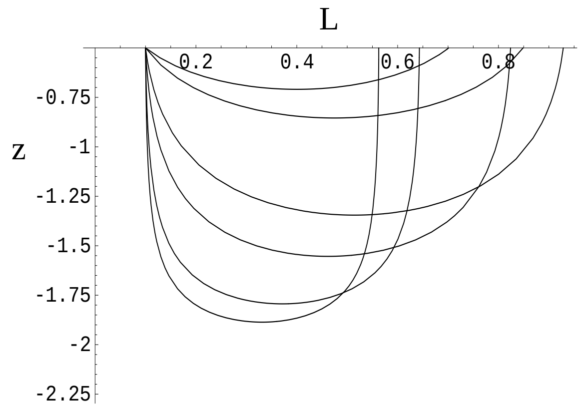

Figure 5: Wilson loops in YM* showing how the depth the probe string penetrates into the deformed space depends on the quark separation (or equivalently the initial condition of z').

to enter the deformed space the behaviour changes. At a critical value of z' the string, although still penetrating deeper into the space, returns to $z = -0.5$ at a *shorter* quark separation. Thus there is a maximum quark separation described by the Euler Lagrange equation solutions.

To get a better handle on this behaviour we can use the x independence of the lagrangian which implies the hamiltonian is conserved

$$\frac{\sqrt{\xi_+}e^{2A}}{\sqrt{1+z'^2+\alpha'^2 e^{-2A}}} = \text{constant}\,. \tag{5.12}$$

Thus, for α constant,

$$z' = \sqrt{\frac{e^{4A}\xi_+}{c^2} - 1}\,. \tag{5.13}$$

We can determine the integration constant c by evaluating the above equation at the point of maximum depth reach by the string, z_0, where $z' = 0$ and which corresponds to $x = 0$ (because of the the symmetry of the problem)

$$c^2 = \xi_+(z_0)e^{4A(z_0)}\,. \tag{5.14}$$

The quark anti-quark separation is then given by

$$\frac{L}{2} = \int_{z_0}^{z_{\max}} dz \frac{1}{\sqrt{\frac{e^{4A}\xi_+}{c^2} - 1}}\,. \tag{5.15}$$

The energy of the string is given by S/T so we find

$$E = \frac{1}{\pi c}\int_{z_0}^{z_{\max}} dz \frac{e^{4A}\xi_+}{\sqrt{\frac{e^{4A}\xi_+}{c^2} - 1}}\,. \tag{5.16}$$

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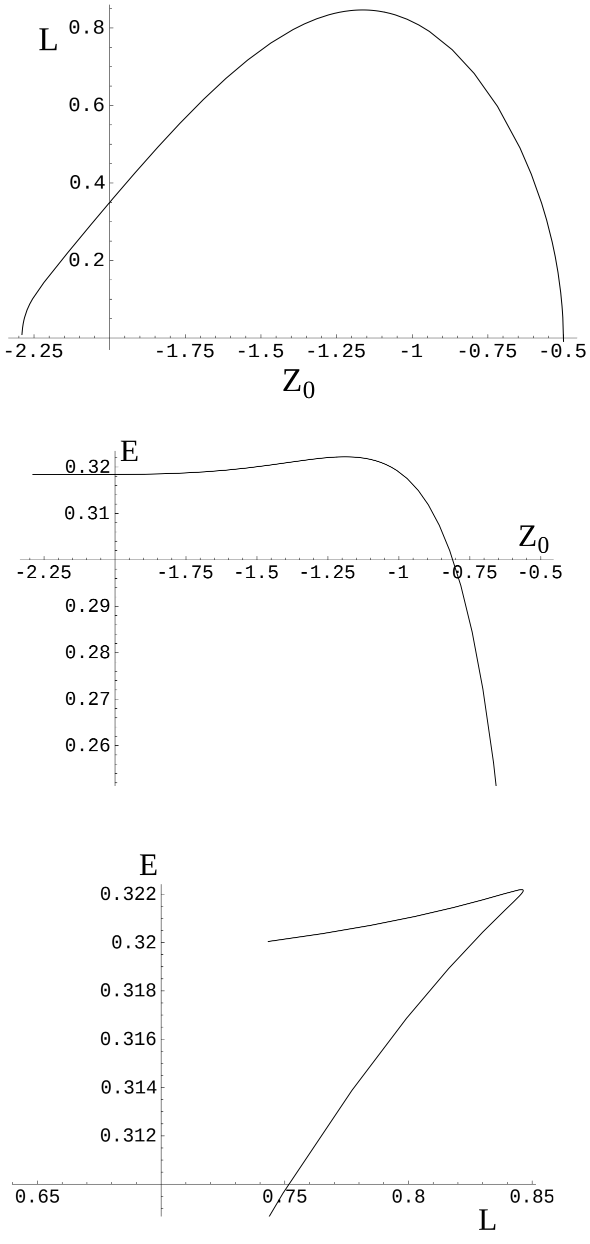

Figure 6: YM* Wilson loop results at $\alpha = \pi/2$; we plot the quark separation L vs the maximum depth z_0 that the string reaches; the energy of the string vs the maximum depth; and finally the energy vs the quark separation.

These equations are again straightforward to solve numerically in the YM* background. In figure 6 we show plots of the quark anti-quark separation vs z_0, the energy of the string vs z_0 and finally the energy of the string vs quark anti-quark separation. We see again the maximum separation but now also that the strings that penetrate into the interior of the deformed space are higher action.

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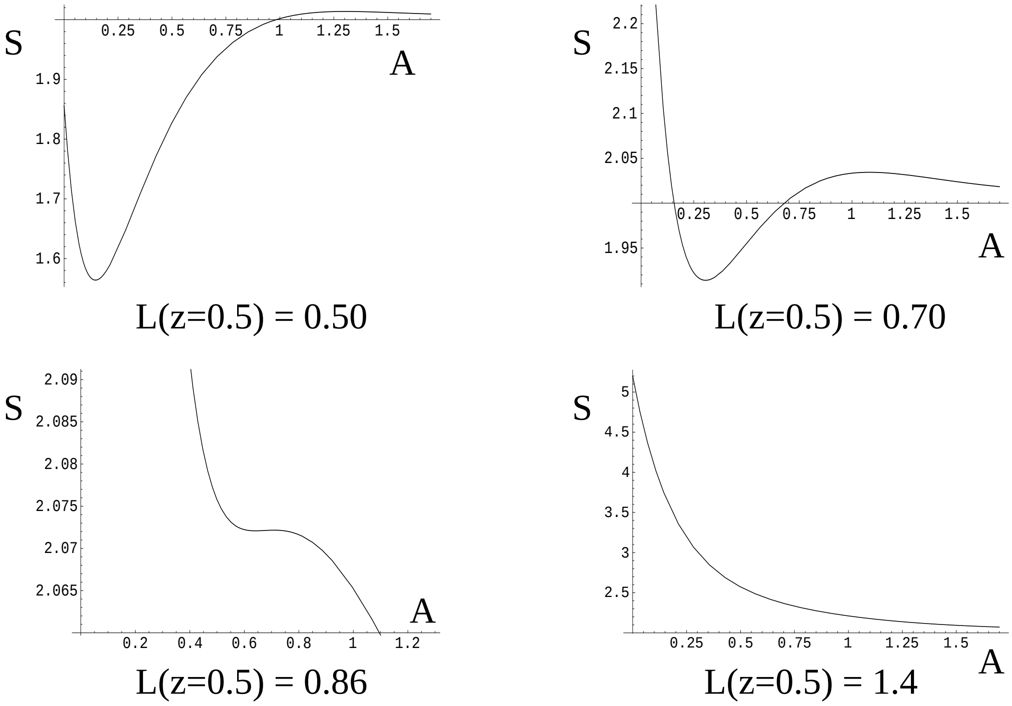

Figure 7: The action of sinusoidal string configurations against amplitude (penetration into the space) for four different values of the quark separation L.

This behaviour naively suggests string breaking by quark anti-quark pair production (an interpretation we put forward in the first version of this paper). We must be careful though since the Euler Lagrange equations only return local turning points of the solution (one of our solutions for fixed separation must therefore be a maximum) and it is possible they miss some run-away direction in the potential. To address this issue we should consider a set of strings linking quarks at fixed separation but which penetrate the space to different degrees. Ideally one should include the true solution of the Euler Lagrange equations in this set but we will learn all we need from a simpler set of configurations. We take just a sinusoidal shape for the string

$$z(x) = -A \sin \frac{\pi x}{L}\,, \tag{5.17}$$

where we vary the amplitude A.

In figure 7 we plot the action of these configurations as a function of A for L smaller than, larger than and close to the critical quark separation found above.

It is now clear what the Euler Lagrange equation solutions correspond to. There is first a minimum of the action corresponding to an AdS-like string, then a maximum (the second solution above) and finally the action falls as the string moves into the singularity at $z = -2.3$. When the quarks are close the global minimum is the AdS-like solution but at a critical separation there is a transition and the preferred configuration is that which falls into the singularity. We have also looked at solutions at fixed $\alpha = 0$ and with varying α. Although there is a little more structure in the α variation the same instability exists as that just described.

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We conclude that we can not understand the behaviour of the Wilson loops at the supergravity level for the IR. One could imagine that with an appropriate stringy resolution of the singularity, perhaps in terms of some fuzzy brane configuration, the situation might be remedied. It could be that the strings prefer to form a bound state with the interior branes and lie along the singularity (the mechanism suggested by Polchinski and Strassler [16]) giving rise to a confining potential but this must remain speculation in these cases.

6. Summary

In this paper we have studied aspects of confinement in two deformed AdS geometries, Yang Mills* and $\mathcal{N} = 1^*$. We have shown that they describe discrete glueball spectra indicating that the geometries do confine. Study of bound states of the massive fermions in the geometries show that there is little decoupling of these massive states. Nevertheless the glueball spectra agree well with other gravity duals with the same infra-red degrees of freedom, even if the ultra-violet of the theories differ significantly.

We also studied the behaviour of Wilson loops in the 10d lift of Yang Mills*. We found though that when a quark pair is separated beyond some critical separation the geometry prefers a string configuration connecting the quarks that falls into the singularity. It is therefore not possible to understand Wilson loop behaviour without a stringy resolution of the interior which currently is lacking.

Acknowledgments

N.E. is grateful to PPARC for the support of an Advanced Fellowship. D.C. is grateful to PPARC for the support of studentships. M.P. is supported by the European Commission Marie Curie Postdoctoral Fellowship under contract number HPRN–CT–2001–01277 and partially by the EU contract HPRN–CT–2000–00122. R.A. was supported by INFN and MURST and at a later stage by a PPARC postdoctoral fellowship. R.A. would like to thank Prof. M. Bianchi and Prof. A. Zaffaroni for very useful discussions and the physics department of Southampton University for kind hospitality.

References

[1] J.M. Maldacena, *The large-N limit of superconformal field theories and supergravity*, *Adv. Theor. Math. Phys.* **2** (1998) 231 [hep-th/9711200].

[2] S.S. Gubser, I.R. Klebanov and A.M. Polyakov, *Gauge theory correlators from non-critical string theory*, *Phys. Lett.* **B 428** (1998) 105 [hep-th/9802109].

[3] E. Witten, *Anti-de Sitter space and holography*, *Adv. Theor. Math. Phys.* **2** (1998) 253 [hep-th/9802150].

[4] E. Witten, *Anti-de Sitter space, thermal phase transition and confinement in gauge theories*, *Adv. Theor. Math. Phys.* **2** (1998) 505 [hep-th/9803131].

JHEP05(2004)065

[5] I.R. Klebanov and M.J. Strassler, *Supergravity and a confining gauge theory: duality cascades and χSB-resolution of naked singularities*, *J. High Energy Phys.* **08** (2000) 052 [hep-th/0007191].

[6] J.M. Maldacena and C. Núñez, *Towards the large-N limit of pure $N = 1$ super Yang-Mills*, *Phys. Rev. Lett.* **86** (2001) 588 [hep-th/0008001].

[7] C. Csáki, H. Ooguri, Y. Oz and J. Terning, *Glueball mass spectrum from supergravity*, *J. High Energy Phys.* **01** (1999) 017 [hep-th/9806021].

[8] J.M. Maldacena, *Wilson loops in large-N field theories*, *Phys. Rev. Lett.* **80** (1998) 4859 [hep-th/9803002].

[9] S.J. Rey and J.T. Yee, *Macroscopic strings as heavy quarks in large-N gauge theory and anti-de Sitter supergravity*, *Eur. Phys. J.* **C 22** (2001) 379 [hep-th/9803001]; *Wilson-Polyakov loop at finite temperature in large-N gauge theory and anti-de Sitter supergravity*, *Nucl. Phys.* **B 527** (1998) 171 [hep-th/9803135].

[10] L. Girardello, M. Petrini, M. Porrati and A. Zaffaroni, *Novel local CFT and exact results on perturbations of $N = 4$ super Yang-Mills from AdS dynamics*, *J. High Energy Phys.* **12** (1998) 022 [hep-th/9810126].

[11] A. Khavaev, K. Pilch and N.P. Warner, *New vacua of gauged $N = 8$ supergravity in five dimensions*, *Phys. Lett.* **B 487** (2000) 14 [hep-th/9812035].

[12] K. Pilch and N.P. Warner, *$N = 1$ supersymmetric renormalization group flows from IIB supergravity*, *Adv. Theor. Math. Phys.* **4** (2002) 627 [hep-th/0006066].

[13] J. Babington, D.E. Crooks and N.J. Evans, *A stable supergravity dual of non-supersymmetric glue*, *Phys. Rev.* **D 67** (2003) 066007 [hep-th/0210068].

[14] L. Girardello, M. Petrini, M. Porrati and A. Zaffaroni, *The supergravity dual of $N = 1$ super Yang-Mills theory*, *Nucl. Phys.* **B 569** (2000) 451 [hep-th/9909047].

[15] M. Petrini and A. Zaffaroni, *The holographic RG flow to conformal and non-conformal theory*, hep-th/0002172.

[16] J. Polchinski and M.J. Strassler, *The string dual of a confining four-dimensional gauge theory*, hep-th/0003136.

[17] S.S. Gubser, *Curvature singularities: the good, the bad and the naked*, *Adv. Theor. Math. Phys.* **4** (2002) 679 [hep-th/0002160].

[18] I. Montvay, *Supersymmetric Yang-Mills theory on the lattice*, *Int. J. Mod. Phys.* **A 17** (2002) 2377 [hep-lat/0112007]; A. Feo, *Supersymmetry on the lattice*, *Nucl. Phys.* **119** *(Proc. Suppl.)* (2003) 198 [hep-lat/0210015].

[19] K. Pilch and N.P. Warner, *$N = 2$ supersymmetric RG flows and the IIB dilaton*, *Nucl. Phys.* **B 594** (2001) 209 [hep-th/0004063].

[20] A. Brandhuber and K. Sfetsos, *An $N = 2$ gauge theory and its supergravity dual*, *Phys. Lett.* **B 488** (2000) 373 [hep-th/0004148].

[21] N.J. Evans and M. Petrini, *AdS RG-flow and the super–Yang-Mills cascade*, *Nucl. Phys.* **B 592** (2001) 129 [hep-th/0006048].

JHEP05(2004)065

[22] A. Buchel, A.W. Peet and J. Polchinski, *Gauge dual and noncommutative extension of an $N = 2$ supergravity solution*, *Phys. Rev.* **D 63** (2001) 044009 [hep-th/0008076].

[23] N.J. Evans, C.V. Johnson and M. Petrini, *The enhancon and $N = 2$ gauge theory/gravity RG flows*, *J. High Energy Phys.* **10** (2000) 022 [hep-th/0008081].

[24] C.V. Johnson, A.W. Peet and J. Polchinski, *Gauge theory and the excision of repulson singularities*, *Phys. Rev.* **D 61** (2000) 086001 [hep-th/9911161].

[25] M. Bianchi, O. DeWolfe, D.Z. Freedman and K. Pilch, *Anatomy of two holographic renormalization group flows*, *J. High Energy Phys.* **01** (2001) 021 [hep-th/0009156].

[26] O. DeWolfe and D.Z. Freedman, *Notes on fluctuations and correlation functions in holographic renormalization group flows*, hep-th/0002226.

[27] G. Arutyunov, S. Frolov and S. Theisen, *A note on gravity-scalar fluctuations in holographic RG flow geometries*, *Phys. Lett.* **B 484** (2000) 295 [hep-th/0003116].

[28] D. Anselmi, L. Girardello, M. Porrati and A. Zaffaroni, *A note on the holographic beta and c functions*, *Phys. Lett.* **B 481** (2000) 346 [hep-th/0002066].

[29] M. Bianchi, D.Z. Freedman and K. Skenderis, *Holographic renormalization*, *Nucl. Phys.* **B 631** (2002) 159 [hep-th/0112119].

[30] M. Henningson and K. Skenderis, *The holographic Weyl anomaly*, *J. High Energy Phys.* **07** (1998) 023 [hep-th/9806087]; *Holography and the Weyl anomaly*, *Fortschr. Phys.* **48** (2000) 125 [hep-th/9812032].

[31] S. de Haro, S.N. Solodukhin and K. Skenderis, *Holographic reconstruction of spacetime and renormalization in the AdS/CFT correspondence*, *Commun. Math. Phys.* **217** (2001) 595 [hep-th/0002230].

[32] M. Bianchi, D.Z. Freedman and K. Skenderis, *How to go with an RG flow*, *J. High Energy Phys.* **08** (2001) 041 [hep-th/0105276].

[33] D.Z. Freedman, S.S. Gubser, K. Pilch and N.P. Warner, *Renormalization group flows from holography supersymmetry and a c-theorem*, *Adv. Theor. Math. Phys.* **3** (1999) 363 [hep-th/9904017].

[34] N. Dorey, *An elliptic superpotential for softly broken $N = 4$ supersymmetric Yang-Mills theory*, *J. High Energy Phys.* **07** (1999) 021 [hep-th/9906011];
N. Dorey and S.P. Kumar, *Softly-broken $N = 4$ supersymmetry in the large-N limit*, *J. High Energy Phys.* **02** (2000) 006 [hep-th/0001103];
O. Aharony, N. Dorey and S.P. Kumar, *New modular invariance in the $N = 1^*$ theory, operator mixings and supergravity singularities*, *J. High Energy Phys.* **06** (2000) 026 [hep-th/0006008].

[35] M. Krasnitz, *A two point function in a cascading $N = 1$ gauge theory from supergravity*, hep-th/0011179.

[36] L. Ametller, J.M. Pons and P. Talavera, *On the consistency of the $N = 1$ SYM spectra from wrapped five-branes*, *Nucl. Phys.* **B 674** (2003) 231 [hep-th/0305075].

[37] H. Boschi-Filho and N.R.F. Braga, *Gauge/string duality and scalar glueball mass ratios*, *J. High Energy Phys.* **05** (2003) 009 [hep-th/0212207]; *QCD/string holographic mapping and glueball mass spectrum*, *Eur. Phys. J.* **C 32** (2004) 529 [hep-th/0209080].

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[38] C.J. Morningstar and M.J. Peardon, *The glueball spectrum from an anisotropic lattice study*, *Phys. Rev.* **D 60** (1999) 034509 [hep-lat/9901004].

[39] B. Lucini and M. Teper, SU(N) *gauge theories in four dimensions: exploring the approach to* $N = \infty$, *J. High Energy Phys.* **06** (2001) 050 [hep-lat/0103027].

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